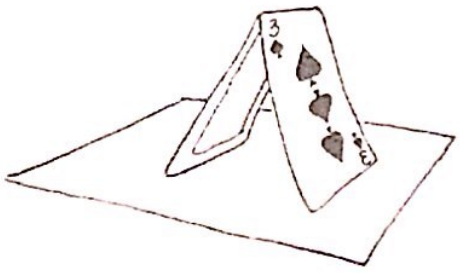
Annabel 11.5

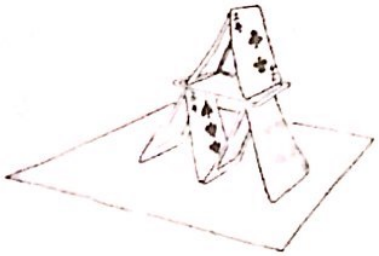
Year 11
Unit 3 Number patterns
Investigating pattern, Communication in mathematics

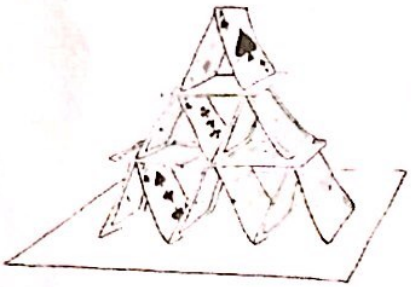
A house of cards

Mike is building a house of cards. For one storey he uses two cards.



For two storeys he uses five more cards; a total of 7 cards as shown in the diagram.





For three storeys, the house of cards looks like this:

How many cards are used?

How many cards would be used in an n storey house of cards? Explain clearly your reasoning and provide a justification for any rule that you find.

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Criterion B: Investigating Patterns

Level	Descriptor
0	I do not reach a standard described by any of the descriptors given below.
1-2	I **apply, with some guidance,** mathematical problem-solving techniques to recognize **simple** patterns.
3-4	I **apply** mathematical problem-solving techniques to recognize patterns, and **suggest** relationships or general rules.
5-6 b	I **select and apply** mathematical problem-solving techniques to recognize patterns, **describe** them as relationships or general rules, and **draw conclusions** consistent with my findings.
7-8	I **select and apply** mathematical problem-solving techniques to recognize patterns, **describe** them as relationships or general rules, **draw conclusions** consistent with my findings, and **provide justifications or proofs.**

Criterion C: Communication in mathematics

Level	Descriptor
0	I do not reach a standard described by any of the descriptors given below.
1-2	I show **basic** use of mathematical language **and/or** forms of mathematical representation but sometimes my lines of reasoning are **difficult to follow.**
3-4	I show **sufficient** use of mathematical language **and** forms of mathematical representation. My lines of reasoning are **clear** though not always **logical** or **complete**. I move between different forms of representation **with some success.**
5-6 b	I show **good** use of mathematical language **and** forms of mathematical representation. My lines of reasoning are **concise, logical** and **complete.** I move **effectively** between different forms of representation.

Criterion B - Investigating Patterns, Communication in Mathematics

Annabel Suen 11.5

A HOUSE OF CARDS.

In this investigation, I will need to find the sequence that explains the amount of cards that would be used in an n storey house of cards. Then predict the number for 5 storeys with the rule and test it out. Lastly, attempt to justify my rule.

1 storey

Total 2 cards

2 storey

Total 7 cards

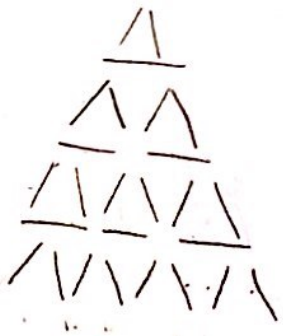
3 storey

Total 15 cards

Finding the rule - Let n be the number of storeys and un be the number of cards used.

n	1	2	3	4
Un	2	7	15	26
1st Diff		5	8	11
2nd Diff			3	3
$1.5n^2$	1.5	6	13.5	24
$Un-1.5n^2$	0.5	1	1.5	2
1st Diff		0.5	0.5	0.5
0.5n	0.5	1	1.5	2
$(un-1.5n^2) - 0.5n$	0	0	0	0

4 storey



Total 26 cards

* I added 4 storey to make it easier to calculate the sequence as 3 terms makes it difficult to identify the pattern.

nth Rule : $1.5n^2 + 0.5n$

or

$$\frac{3}{2}n^2 + \frac{1}{2}n$$

Prediction and Test

5 storeys :

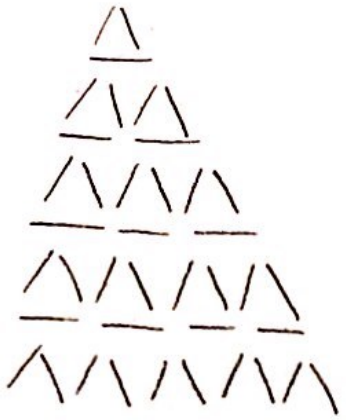
Prediction : $1.5n^2 + 0.5n$

$= 1.5(5)^2 + 0.5(5)$

$= 37.5 + 2.5$

$= 40$ ← prediction

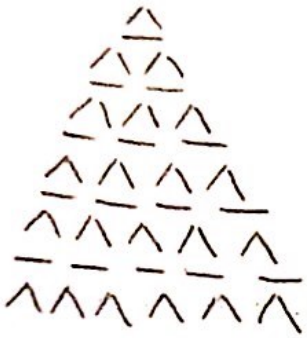
Test.



Total 40 cards.

This rule can be proven by 5 storeys.

Further Test :



Prediction

$1.5(6)^2 + 0.5(6)$

$= 54 + 3$

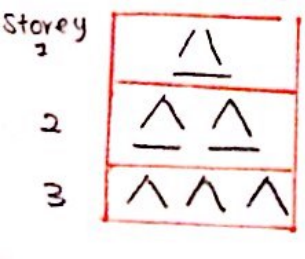
$= 57$ ← prediction

Total 57 cards

This rule is proven by the next 2 terms in the pattern.

Justification Let n be the number of the storey

This rule works as for every storey (except the most bottom storey of the pattern), the number of cards used to construct that storey (un) is 3 x the number of the storey ($3n$). For the most bottom layer/storey of each pattern, it is only 2 x the number of the storey ($2n$). See below.



Total 3 (3×1)

Total 6 (3×2)

Total 6 (2×3)

(2 as sides, 1 as base)

This is because 3 cards form a triangle, and each layer below always have to have 1 more triangle than the storey above in order for [illegible] in the bottom layer as the table will act as it's base. Hence it does not need [illegible]