Year 11
Unit 1 – Off the straight and narrow
Investigating pattern

Calculate the following:

$$(24\times 22)-(33\times 13)$$
$$(24+22)+(33+13)$$
$$(33-13)-(24-22)$$

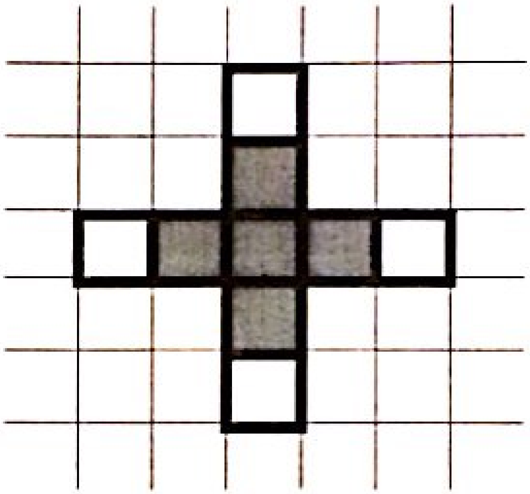
Investigate what happens to these calculations as the cross moves around the grid.

Write down any rules that you find, and explain why they work.

The crosses investigated above were 1-arm crosses.

Shown opposite is a 2-arm cross.

Investigate for Cross Numbers of different sizes, and state any limitations of your rule(s).



Criterion B: Investigating Patterns

Level	*Descriptor*
0	*I do not reach a standard described by any of the descriptors given below.*
1-2	***I apply, with some guidance,*** *mathematical problem-solving techniques to recognize* ***simple*** *patterns.*
3-4	***I select and apply*** *mathematical problem-solving techniques to recognize patterns, and* ***suggest*** *relationships or general rules.*
5-6	***I select and apply*** *mathematical problem-solving techniques to recognize patterns,* ***describe*** *them as relationships or general rules, and* ***draw conclusions*** *consistent with my findings.*
7-8	***I select and apply*** *mathematical problem-solving techniques to recognize patterns,* ***describe*** *them as relationships or general rules,* ***draw conclusions*** *consistent with my findings, and* ***provide justifications or proofs.***

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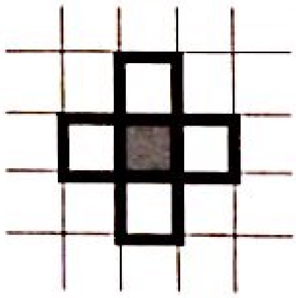
Cross numbers

Shown below is a number square which contains the numbers 1 to 100:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

We can form **Cross Numbers** by placing a cross on top of this grid.

We could use a cross like the one shown opposite:



Here is one example:

1	2	3	4	5
11	12	13	14	15
21	22	23	24	25
31	32	33	34	35
41	42	43	44	45

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Year 11 Unit 1 – Off the straight and narrow.
Criterion B – Investigating Patterns

Our task for this investigation is to find any rules to the value of the numbers inside the cross placed on a 1 to 100 grid.

Example 1 –

	13 (c)	
22 (a)	23 (x)	24 (b)
	33 (d)	

Calculations:

$(22 \times 24) - (13 \times 33) = 99$

$(24 + 22) + (13 + 33) = 92$

$(33 - 13) - (24 - 22) = 18$

Example 2 – using a different set of numbers

	5	
14	15	16
	25	

$(14 \times 16) - (5 \times 25) = 99$

$(14 + 16) + (5 + 25) = 60$

$(25 - 5) - (16 - 14) = 18$

Example 3 –

	39	
48	49	50
	59	

$(48 \times 50) - (39 \times 59) = 99$

$(48 + 50) + (39 + 59) = 196$

$(59 - 39) - (50 - 48) = 18$

Results.	Middle number	Equation 1	Equation 2	Equation 3
Example 1	23	99	92	18
Example 2	15	99	60	18
Example 3	49	99	196	18

From the results above, I see that the answers for equation 1 all result in 99, equation 2 results to $4x$ (let the centre number of the cross be x) and equation 3 results to 18. From this, I can predict 3 rule that would work for which ever number.

a) $(x-1)(x+1) - (x-10)(x+10) = (x^2-1) - (x^2-100) = 99$

b) $[(x+1)+(x-1)] + [(x+10)+(x-10)] = 4x$

c) $[(x+10)-(x-10)] - [(x+1)-(x-1)] = 18$

	$x-10$	
$x-1$	x	$x+1$
	$x+10$	

Rule A

$[(x+1)(x-1)] - [(x+10)(x-10)]$

$(x^2-1) - (x^2-100)$

$-1 + 100 = 99$

Rule B

$(x+1)+(x-1)+(x+10)+(x-10)$

$= 4x$

Why does these rules work?

Rule A $[(x^2-1^2)-(x^2-10^2)]$ works for any number as it is the same number on both side of the equation no matter what number x represents. Hence, it does not break the ratio and the balance of the equation and it would result to 99. It uses the quadratic law $(a+b)(a-b) = (a^2-b^2)$.

Rule B $[(x-1)+(x+1)+(x-10)+(x+10)]$ also works to result into $4x$ as the result is proportional to whichever number x represents.

Rule C $[(x+10)-(x-10)]-[(x+1)-(x-1)]$ always results to 18 as x is the same value on both sides.

Trying with a different size cross.

Example 4.

		3		
		13		
21	22	23	24	25
		33		
		43		

$(21 \times 25) - (3 \times 43) = 396$

$(21 + 25) + (3 + 43) = 92$

$(43 - 3) - (25 - 21) = 36$

Example 5. [25]

$(23 \times 27) - (5 \times 45) = 396$

$(23 + 27) + (5 + 45) = 100$

$(45 - 5) - (27 - 23) = 36$

Example 6 [78]

$(76 \times 80) - (58 \times 98) = 396$

$(76 + 80) - (58 + 98) = 312$

$(98 - 58) - (80 - 76) = 36$

Results	Middle number	Equation 1	Equation 2	Equation 3
Example 4	23	396	396	396
Example 5	25	92	100	312.
Example 6	78	36	36	36

Final rules.

From my 2 experiment above, I can conclude these rules. Let a be the number of boxes away from the centre number.

a) $(x^2$ − number of boxes away (a) from the centre number$^2)$ −

$(x^2$ + number of boxes (a) away from the centre number $\times 10^2)$

$= [(x-a)(x+a)]^2 - [(x-10a)(x+10a)]$

Rule B) $[(x-a)+(x+a) + (x-10a)+ (x+10a)]$ x= centre numbre.

C) $[(x+10a)-(x-10a)]-[(x+a)-(x-a)]$ a = number of boxes away from the centre.

Simplify

These rules are effective on crosses of all sizes of crosses, but there are limitation to it.

If any of the x values used in the centre of the cross would result in a negative value at any part of the equations, the equation would become ineffective.

Final relationship between crosses.

	equation 1	equation 2	equation 3
1 leg cross	99	$4x$	18
2 leg cross	396	$4x$	36
3 leg cross	891	$4x$	54
a leg cross	99a	$4x$	18a

Justification

- Work on it with another cross. (e.g. 4 leg cross)
- First predict the answer with the equation above ^
- The recalculate from the three equations to prove your answer

- State limitations
  - Only works if a is less then 5 as the cross would not fit on the 1-100 grid

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